Weighted Clique and Independent Set in Edge-Distant Hereditary Graphs

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In [1], Borowiecki, Drgas-Burchardt, and Sidorowicz studied the k-apex class of a hereditary graph class $\mathcal{G}$, consisting of graphs that are at most k vertex deletions away from $\mathcal{G}$. In [2, 3], Singh and Sivaraman extended this concept to the p-edge-apex and q-edge-add classes of a hereditary class $\mathcal{G}$, consisting of graphs that are at most p edge deletions and q non-edge additions away from $\mathcal{G}$, respectively. They showed that the p-edge-apex and q-edge-add classes of a hereditary graph class $\mathcal{G}$ are themselves hereditary, and proved that they have finite forbidden induced subgraphs whenever $\mathcal{G}$ has finite forbidden induced subgraphs.

In this work, we investigate the time complexity of the *Weighted Maximum Clique Problem* (WMCP) and the *Weighted Maximum Independent Set Problem* (WMISP) when restricted to the p-edge-apex and q-edge-add classes of a hereditary graph class $\mathcal{G}$. In this setting, we introduce a parameter $\xi_{\mathcal{G}}$, called the *$\mathcal{G}$-edge distance* of a graph G, defined as the minimum integer k such that G can be transformed into a graph in $\mathcal{G}$ by either deleting k edges or adding k non-edges.

Using this parameter, we present an $O(2^k \cdot f(m,n))$-time algorithm for both WMCP and WMISP. The input to the algorithm consists of a graph G with $\xi_{\mathcal{G}}(G) = k$ and a set S of edges or non-edges such that either $G - S \in \mathcal{G}$ or $G + S \in \mathcal{G}$, respectively, assuming that WMCP and WMISP can be solved in $O(f(m,n))$ time when restricted to the class $\mathcal{G}$.

A graph G with vertex set $V(G)$ and edge set $E(G)$ is said to be *word-representable* if there exists a word w over the alphabet $V(G)$ such that, for any two distinct letters $x, y \in V(G)$, the letters x and y alternate in w if and only if $xy \in E(G)$. The main motivation to conduct the above study is as follows:

- It is known that the *Weighted Maximum Clique Problem* (WMCP) is solvable in polynomial time for the class of word-representable graphs. However, the tractability of the *Weighted Maximum Independent Set Problem* (WMISP) for word-representable graph classes remains unknown.
- Interestingly, both the *Weighted Maximum Clique Problem* (WMCP) and the *Weighted Maximum Independent Set Problem* (WMISP) are solvable in polynomial time on the classes of comparability graphs and circle graphs. Moreover, both comparability graphs and circle graphs form proper subclasses of the class of word-representable graphs. Consequently, the hereditary nature of these two classes motivates the application of the above algorithmic framework to study the time complexity of WMISP on word-representable graphs, parameterized by either the comparability-edge distance or the circle-edge distance.
- Another interesting direction for future research is to treat the $\mathcal{G}$-edge distance analogously to classical graph parameters such as treewidth and cliquewidth, and to investigate bounds on the $\mathcal{G}$-edge distance for various graph classes.
- Finally, it would be interesting to study other computationally hard problems and to develop parameterized algorithms for them using the $\mathcal{G}$-edge distance as a parameter.

The algorithm normally requires two inputs: the graph G and a set S of edges or non-edges such that removing or adding S places the graph in the class $\mathcal{G}$. However, for word-representable graphs, we can do better. In this case, it is sufficient to provide the algorithm with the graph and a word that represents it. From this word, the required set S can be computed in polynomial time. As a result, this algorithmic framework becomes particularly interesting and worthy of further study.

References

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